10/May/23

Chapter = 3

Trignometric functions

Exercise - 3.1

Q1- Find - - - - - - - - - - - - - measures:-

π Radian $= 180^\circ$

1 Radian $= \left[\frac{180}{\pi}\right]^\circ$

$1^\circ = \left[\frac{\pi}{180}\right]$ rad

$1^\circ = 60'$

$1' = 60''$

(i) 25°

$$25^\circ = \cancel{25}^{5} \times \frac{\pi}{\cancel{180}_{36}} \text{ rad}$$

$$= \frac{5\pi}{36} \text{ rad}$$

(ii) $-47^\circ 30'$

$$= -\left[47^\circ + \left(\frac{30}{60}\right)^\circ\right]$$

$$= -\left[47^\circ + \frac{1}{2}^\circ\right]$$

$$= -\left[\frac{94+1}{2}\right]^\circ$$

$$= -\frac{95^\circ}{2}$$

$$= -\frac{\cancel{95}^{19}}{2} \times \frac{\pi}{\cancel{180}_{36}} = -\frac{19\pi}{72} \text{ radian}$$

(iii) $240°$

$$\cancel{240}^{\cancel{12}\,4} \times \frac{\pi}{\cancel{180}_{\cancel{9}\,3}}$$

$$= \frac{4\pi}{3} \text{ radian}$$

(iv) $520°$

$$\cancel{520}^{26} \times \frac{\pi}{\cancel{180}_{9}}$$

$$= \frac{26\pi}{9} \text{ radian}$$

Ques2- find - - - - - - - - - - - - measures

(i) $\frac{11}{16}$ rad $\rightarrow (\ \ )°$

$$\frac{11}{16} \text{ rad} = \left(\frac{11}{16} \times \frac{180}{\pi}\right)°$$

$$= \left[\frac{\cancel{11}}{\cancel{16}_{8}} \times \frac{\cancel{180}^{\cancel{90}\,45} \times 7}{\cancel{22}_{2}}\right]°$$

$$= \left[\frac{315}{8}\right]°$$

$$= \quad 39° + \frac{3}{8}°$$

$$= 39° + \frac{3}{\cancel{8}_2} \times \cancel{360}^{15}$$

$$= 39° + \frac{45'}{2}$$

$$= 39° + 22' + \frac{1'}{2}$$

$$= 39°22' + \frac{1}{\cancel{2}} \times \cancel{60}^{30}$$

$$= 39°22'30''$$

Q3- A - - - - - - - - - - - - - - - second?

No. of revolution in one min (60 sec.) = 360

" " 1 sec. $= \frac{360}{60} = 6$

Angle in 1 revolution $= 2\pi$

" " 6 " $= 6 \times 2\pi$

$= 12\pi$

Q5- In a circle - - - - - - - - - - - chord.

$d = 40$ cm

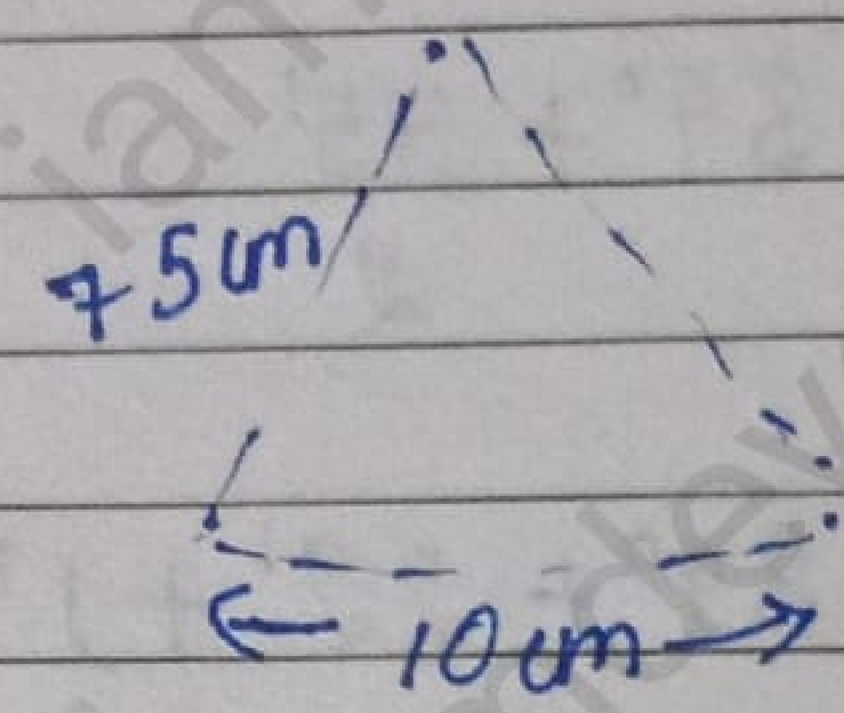
$l = 20$ cm $\qquad l = r\theta$

ΔOAB is eq. $\Delta = 60°$ $\qquad = 60 \times \frac{\pi}{\cancel{180}_3}$

$= 20 \times \frac{\pi}{3}$ (Ans) $\qquad = \frac{\pi}{3}$

Ques7 - find - - - - - - - - - length

(i) 10cm



$l = 10\,cm$

$l = r\theta \Rightarrow \theta = \frac{l}{r}$

$\theta = \frac{\cancel{10}\,2}{\cancel{75}\,15}$

$\theta = \frac{2}{15}$ rad (Ans)

(ii) 15cm

$l = 15cm$

$\theta = \frac{\cancel{15}\,1}{\cancel{75}\,5}$

$\theta = \frac{1}{5}$ rad.

(iii) 21cm

$\theta = \frac{\cancel{21}\,7}{\cancel{75}\,25}$

Q2 (ii) -4

$$\frac{-4\times180}{\pi} = -\cancel{4}^{2}\times\frac{180\times7}{\cancel{22}_{11}}$$

$$= \frac{-14\times180}{11} = \frac{-2520}{11} \text{ degree}$$

$$\frac{-2520}{11} = -229\frac{1}{11}^{\circ} = -229^{\circ} + \frac{1}{11}\times60'$$

$$\Rightarrow -229^{\circ} + 5^{\circ} + 5\times60''$$

$$= -229^{\circ}\,5'\,27''$$

(iii) $5\frac{\cancel{\pi}}{\cancel{3}}\times\frac{\cancel{180}^{60}}{\cancel{\pi}} = 300^{\circ}$

(iv) $\frac{7\cancel{\pi}}{\cancel{6}}\times\frac{\cancel{180}^{30}}{\cancel{\pi}} = 210^{\circ}$

Q4. find . . . - - - - - - - - - - 22 cm

$l = 22\text{ cm}$ $r = 100\text{ cm}$

$\theta = \frac{l}{r}$ $\theta = \frac{22}{100}$ radian

$\theta = \frac{22}{10\cancel{0}}\times\frac{18\cancel{0}}{\pi} \Rightarrow \theta = \frac{\cancel{22}}{10}\times\frac{18\times7}{\cancel{22}} \Rightarrow \frac{126}{10} \Rightarrow 12\cdot6^{\circ}$

Ques 6 - If . - - - - - - - - - - - radii

$\theta_1 = 60^{\circ}$ $\theta_2 = 75^{\circ}$

$r_1 = \frac{l}{\theta_1}$ $r_2 = \frac{l}{\theta_2}$

$r_1 = \frac{l}{60}$ $r_2 = \frac{l}{75}$

$\frac{r_1}{r_2} = \frac{\cancel{l}}{60}\times\frac{75}{\cancel{l}} = \frac{\cancel{75}}{\cancel{60}}\ \frac{}{20\ 4} = \frac{5}{4}$ $r_1 : r_2 = 5:4$

Exercise = 3.2

Q1: Find - - - - - - - - - - - - - - functions

1. $\cos x = -\frac{1}{2}$ x lies in III quadrant.

$$\sin^2 x = 1 - \cos^2 x$$
$$= 1 - \left(-\frac{1}{2}\right)^2$$
$$= 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$$
$$\sin^2 x = \frac{3}{4}$$
$$\sin x = \pm\sqrt{\frac{3}{4}}$$
$$\sin x = -\frac{\sqrt{3}}{2}$$

$$\operatorname{cosec} x = \frac{1}{\sin x}$$
$$\operatorname{cosec} x = -\frac{2}{\sqrt{3}}$$ Ans (2)

$$\sec x = \frac{1}{\cos x}$$
$$\sec x = -2$$ Ans (3)

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$$

$$\boxed{\tan x = \sqrt{3}}$$

Ans (4)

$$\cot x = \frac{1}{\tan x}$$

$$\cot x = \frac{1}{\sqrt{3}}$$

9. $\sin x = \frac{3}{5}$, x lies in II^{nd} quadrant

$\sin x = \frac{3}{5}$ $\quad x \in II^{nd}$ quadrant

$$\cos^2 x = 1 - \sin^2 x$$

$$\cos^2 x = 1 - \left(\frac{3}{5}\right)^2$$

$$\cos^2 x - 1 = 1 - \frac{9}{25}$$

$$= \frac{25-9}{25} = \frac{16}{25}$$

$$\cos x = \pm\sqrt{\frac{16}{25}}$$

$$\cos x = -\frac{4}{5}$$ Ans 1

$$\sec x = \frac{1}{\cos x}$$

$$= \frac{5}{-4}$$ Ans 2

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$= \frac{5}{3}$$ Ans 3

$\tan x = \frac{\sin x}{\cos x}$

$= \frac{3/5}{-4/5}$

$= \frac{3}{-4}$ Ans (4)

$\cot x = \frac{1}{\tan x} \Rightarrow \frac{-4}{3}$ Ans (5)

3. $\cot x = \frac{3}{4}$, x lies in III quadrant

$\cot x = \frac{3}{4}$ $\quad x \in$ IIIrd quadrant

$\tan x = \frac{1}{\cot x}$

$= \frac{4}{3}$ Ans (1)

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\operatorname{cosec}^2\theta = 1 + \frac{9}{16}$

$\operatorname{cosec}^2\theta = \frac{16+9}{16}$

$\operatorname{cosec}\theta = \pm\sqrt{\frac{25}{16}}$

$\operatorname{cosec}\theta = \frac{-5}{4}$ Ans $\quad x \in$ IIIrd quadrant

$\sin x = \frac{1}{\operatorname{cosec} x} = \frac{-4}{5}$ Ans

$$\cos^2 x = 1 - \sin^2 x$$

$$\cos^2 x = 1 - \frac{16}{25}$$

$$\cos^2 x = \frac{25-16}{25}$$

$$\cos^2 x = \frac{9}{25}$$

$$\cos x = \pm\sqrt{\frac{9}{25}}$$

$$\cos x = -\frac{3}{5} \ \underline{\text{Ans}}$$

$$\sec x = \frac{1}{\cos x} = -\frac{5}{3} \ \underline{\text{Ans}}$$

4. $\sec x = \frac{13}{5}$, x lies in fourth quadrant

$$\cos x = \frac{1}{\sec x} = \frac{5}{13} \ \underline{\text{Ans}}$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin^2 x = 1 - \frac{25}{169}$$

$$\sin^2 x = \frac{169-25}{169}$$

$$\sin^2 x = \frac{144}{169}$$

$$\sin x = \pm\sqrt{\frac{144}{169}}$$

$$\sin x = -\frac{12}{13} \quad \therefore x \in \overline{\text{IV}} \text{ quadrant (Ans)}$$

$$\tan x = \frac{-12/\cancel{13}}{5/\cancel{13}}$$

$$\tan x = \frac{-12}{5} \text{ (Ans)}$$

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$\operatorname{cosec} x = \frac{-13}{12} \text{ (Ans)}$$

$$\cot x = \frac{1}{\tan x} \Rightarrow \frac{-5}{12} \text{ (Ans)}$$

5. $\tan x = \frac{-5}{12}$, x lies in II^{nd} quadrant.

$$\sec^2 x = 1 + \tan^2 x$$

$$\sec^2 x = 1 + \left[\frac{25}{144}\right]$$

$$\sec^2 x = \frac{144+25}{144}$$

$$\sec x = \sqrt{\frac{169}{144}}$$

$$\sec x = \pm \frac{13}{12} \text{ Ans } 1$$

$$\cos x = \frac{1}{\sec x}$$

$$= \frac{1}{-\left[\frac{13}{12}\right]} = \frac{-12}{13} \text{ Ans } 2$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{-5}{12} = \frac{\sin x}{\frac{-12}{13}}$$

$$\sin x = -\frac{5}{\cancel{12}} \times \frac{-\cancel{12}}{13} = \frac{5}{13} \text{ Ans } 3$$

$$\operatorname{Cosec} x = \frac{1}{\sin x} = \frac{13}{5} \text{ Ans } (4)$$

$$\cot x = \frac{1}{\tan x}$$

$$= \frac{1}{\left[\frac{-5}{12}\right]}$$

$$= \frac{-12}{5} \text{ Ans } (5)$$

6. $\sin 765°$

$$\sin 765° = \sin[2 \times 360° + 45°]$$
$$\sin 45° = \frac{1}{\sqrt{2}} \text{ (Ans)}$$

7. $\operatorname{cosec}(-1410°)$

$$-\operatorname{cosec} 1410°$$
$$-\operatorname{cosec}[4 \times 360 + (-30°)]$$
$$-\operatorname{cosec}(-30°)$$
$$\operatorname{cosec} 30° = 2 \text{ (Ans)}$$

~~8~~9. $\sin\left[\frac{-11\pi}{3}\right]$

$$-\sin\left[\frac{11\pi}{3}\right]$$

$$= -\sin\left[4\pi - \frac{\pi}{3}\right] = -\left[-\sin\frac{\pi}{3}\right] = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \text{ (Ans)}$$

8. $\tan \frac{19\pi}{3}$

$\tan\left[6\pi + \frac{\pi}{3}\right] = \tan\frac{\pi}{3}$

$= \tan 60^\circ$

$= \sqrt{3}$ (Ans)

10. $\cot\left[\frac{-15\pi}{4}\right]$

$\cot\left[\frac{-15\pi}{4} + 4\pi\right]$

$= \cot\frac{\pi}{4} = 1$ (Ans)

Exercise = 3.3
Formulas

Case 1:- $\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

$\cot(-x) = -\cot x$

$\sec(-x) = \sec x$

$\operatorname{cosec}(-x) = -\operatorname{cosec} x$

Case 2:- $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$$

$$\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x = 2\cos^2 x - 1$

$$\tan^2 x = \frac{2\tan x}{1 - \tan^2 x}$$

$\sin 3x = 3\sin x - 4\sin^3 x$

$\cos 3x = 4\cos^3 x - 3\cos x$

$$\tan 3x = \frac{3\tan x - \tan 3x}{1 - 3\tan^2 x}$$

* $\cos x + \cos y = 2\cos \frac{x+y}{2} ; \cos \frac{x-y}{2}$

* $\cos x - \cos y = -2\sin \frac{x+y}{2} , \sin \frac{x-y}{2}$

* $\sin x + \sin y = 2\sin \frac{x+y}{2} , \cos \frac{x-y}{2}$

* $\sin x - \sin y = 2\cos \frac{x+y}{2} , \sin \frac{x-y}{2}$

* $2\cos x \cos y = \cos(x+y) + \cos(x-y)$
* $-2\sin x \sin y = \cos(x+y) - \cos(x-y)$
* $2\sin x \cos y = \sin(x+y) + \sin(x+y)$
* $2\cos x \sin y = \sin(x+y) - \sin(x-y)$

Exercise - 3.3

1. $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$.

L.H.S. $= \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4}$

$= \left[\frac{1}{2}\right]^2 + \left[\frac{1}{2}\right]^2 - (1)^2$

$= \frac{1}{4} + \frac{1}{4} - 1$

$= \frac{1+1-4}{4} = \frac{2-4}{4} = \frac{-2}{4} = \frac{-1}{2} = \underline{R.H.S}$

L.H.S. = R.H.S. Hence proved

4. $2\sin^2 \frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

L.H.S. $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$= 2\sin^2\left[\pi - \frac{\pi}{4}\right] + 2\times\left[\frac{1}{\sqrt{2}}\right]^2 + 2\times(2)^2$

$= 2\sin^2\frac{\pi}{4} + \cancel{2}\times\frac{1}{\cancel{2}} + 2\times 4$

$= \left[2\times\frac{1}{\sqrt{2}}\right]^2 + 1 + 8$

$= \cancel{2}\times\frac{1}{\cancel{2}} + 9 = 1 + 9 = 10$ (Ans)

5. (a) $\sin 75^\circ$

$\sin(45^\circ + 30^\circ)$

$\sin 45^\circ\cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$= \left[\frac{1}{\sqrt{2}}\right]\left[\frac{\sqrt{3}}{2}\right] + \left[\frac{1}{\sqrt{2}}\right]\left[\frac{1}{2}\right]$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$ Ans

(b) $\tan 15^\circ$

$\tan(45^\circ - 30^\circ)$

$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$

$$\frac{1-\frac{1}{\sqrt{3}}}{1+1\left[\frac{1}{\sqrt{3}}\right]} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{3+1-2\sqrt{3}}{(\sqrt{3})^2-(1)^2} = \frac{4-2\sqrt{3}}{3-1} = 2-\sqrt{3}$$ (Ans)

2. $2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$

$$\text{L.H.S.} = 2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$$

$$= 2\left[\frac{1}{2}\right]^2 + \operatorname{cosec}^2\left[\pi+\frac{\pi}{6}\right]\left[\frac{1}{2}\right]^2$$

$$= 2\times\frac{1}{4} + \left[-\operatorname{cosec}\frac{\pi}{6}\right]^2\left[\frac{1}{4}\right]$$

$$= \frac{1}{2} + (-2)^2\left[\frac{1}{4}\right]$$

$$= \frac{1}{2} + \frac{4}{1}\times\frac{1}{4}$$

$$= \frac{1}{2} + \frac{4}{4}$$

$$= \frac{1}{2} + 1$$

$$= \frac{1+2}{2} = \frac{3}{2}$$

L.H.S. = R.H.S.

3. $\cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6} = 6$

$\text{L.H.S.} = \cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6} = 6$

$= (\sqrt{3})^2 + \operatorname{cosec}\left[\pi - \frac{\pi}{6}\right] + 3\left[\frac{1}{\sqrt{3}}\right]^2$

$= 3 + \operatorname{cosec} \frac{\pi}{6} + 3 \times \frac{1}{3}$

$= 3 + 2 \times 1$

$= 6$

L.H.S. = R.H.S.

6. $\cos\left[\frac{\pi}{4} - x\right] \cos\left[\frac{\pi}{4} - y\right] - \sin\left[\frac{\pi}{4} - x\right]\left[\sin \frac{\pi}{4} - y\right] = \sin(x+y)$

$\text{L.H.S.} = \cos\left[\frac{\pi}{4} - x\right] \cos\left[\frac{\pi}{4} - y\right] - \sin\left[\frac{\pi}{4} - x\right] \sin\left[\frac{\pi}{4} - y\right]$

$= \frac{1}{2}\left[2\cos\left[\frac{\pi}{4} - x\right] \cos\left[\frac{\pi}{4} - y\right] + \frac{1}{2}\left[-2\sin\left[\frac{\pi}{4} - x\right] \sin\left[\frac{\pi}{4} - y\right]\right.\right.$

$= \frac{1}{2}\left[\cos\left\{\left[\frac{\pi}{4} - x\right] + \left[\frac{\pi}{4} - y\right]\right\} + \cos\left\{\left[\frac{\pi}{4} - x\right] - \left[\frac{\pi}{4} - y\right]\right\}\right]$

$= \frac{1}{2}\left[\cos\left\{\left[\frac{\pi}{4} - x\right] + \left[\frac{\pi}{4} - y\right]\right\} - \cos\left\{\left[\frac{\pi}{4} - x\right] - \left[\frac{\pi}{4} - y\right]\right\}\right]$

Using the formula

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$-2\sin A \sin B = \cos(A+B) - \cos(A-B)$

$= 2 \times \frac{1}{2}\left[\cos\left\{\left[\frac{\pi}{4} - x\right] + \left[\frac{\pi}{4} - y\right]\right\}\right]$

$$= \cos\left[\frac{\pi}{2} - (x+y)\right]$$

$$= \sin(x+y) = R.H.S.$$

$$L.H.S. = R.H.S.$$

Hence proved

7. $$\frac{\tan\left[\frac{\pi}{4}+x\right]}{\tan\left[\frac{\pi}{4}-x\right]} = \left[\frac{1+\tan x}{1-\tan x}\right]^2$$

$$L.H.S. = \frac{\tan\left[\frac{\pi}{4}+x\right]}{\tan\left[\frac{\pi}{4}-x\right]}$$

By the formula

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$$

$$\frac{\left[\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right]}{\left[\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]} = \frac{\left[\dfrac{1+\tan x}{1-\tan x}\right]}{\left[\dfrac{1-\tan x}{1+\tan x}\right]} = \left[\frac{1+\tan x}{1-\tan x}\right]^2$$

$$L.H.S. = R.H.S.$$

Hence proved

8. $$\frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left[\frac{\pi}{2} + x\right]} = \cot^2 x$$

$$\frac{-\cos x \times \cancel{8}\cos x}{\sin x \times -\sin x}$$

$$= \frac{\cancel{-}\cos^2 x}{\cancel{-}\sin^2 x}$$

$$= \cot^2 x = R.H.S.$$

$$L.H.S. = R.H.S.$$

9. $$\cos\left[\frac{3\pi}{2} + x\right]\cos\left[2\pi + x\right]\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right] = 1$$

$$= \sin x \cdot \cos x \left[\tan x + \cot x\right]$$

$$= \sin x \cdot \cos x \left[\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right]$$

$$= \cancel{\sin x \cos x}\left[\frac{\sin^2 x + \cos^2 x}{\cancel{\cos x \sin x}}\right]$$

$$= 1 = R.H.S.$$

11. $\cos\left[\frac{3\pi}{4}+x\right]-\cos\left[\frac{3\pi}{4}-x\right]=-\sqrt{2}\sin x$

$$-2\sin\frac{\frac{3\pi}{4}+x+\frac{3\pi}{4}-x}{2}\cdot\sin\frac{\frac{3\pi}{4}+x-\frac{3\pi}{4}+x}{2}$$

$$=-2\sin\frac{2\times\frac{3\pi}{4}}{2}\cdot\sin\frac{2x}{2}$$

$$=-2\sin\frac{3\pi}{4}\cdot\sin x$$

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\cdot\sin x$$

$$=-2\sin\frac{\pi}{4}\cdot\sin x$$

$$=-\overset{\sqrt{2}}{\cancel{2}}\times\frac{1}{\cancel{\sqrt{2}}}\cdot\sin x$$

$$=\sqrt{2}\sin x$$

12. $\sin^2 6x-\sin^2 4x=\sin 2x\sin 10x$

$$=\frac{1}{2}\left[2\sin^2 6x-2\sin^2 4x\right]$$

$\cos 2x=1-2\sin^2 x$
$2\sin^2 x=1-\cos 2x$

$$=\frac{1}{2}\left[(\cancel{1}-\cos 12x)-(\cancel{1}-\cos 8x)\right]$$

$$=\frac{1}{2}\left[\cos 8x-\cos 12x\right]$$

$$=\frac{1}{\cancel{2}}\times\cancel{2}\sin\frac{8x+12x}{2}\cdot\sin\frac{12x-8x}{2}$$

$$=\sin 10x\cdot\sin 2x\ \text{R.H.S.}$$

15. $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$

L.H.S $\cot 4x[\sin 5x + \sin 3x]$

$= \cot 4x\left[2\sin\frac{5x+3x}{2} \cdot \cos\frac{5x-3x}{2}\right]$

$= \frac{\cos 4x}{\sin 4x}\left[2\sin 4x \cdot \cos x\right]$

$= 2\cos 4x \cdot \cos x$ — (1)

R.H.S.

$\cot x(\sin 5x - \sin 3x)$

$= \cot x\left(2\sin\frac{5x-3x}{2} \cdot \cos\frac{5x+3x}{2}\right]$

$= \frac{\cos x}{\sin x}\left[2\sin x \cdot \cos 4x\right]$

$= 2\cos 4x \cdot \cos x$

20. $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

L.H.S. $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

$= \dfrac{\cancel{-}[\sin 3x - \sin x]}{\cancel{-}[\cos^2 x - \sin^2 x]}$

$= \dfrac{2\cos\frac{3x+x}{2} \cdot \sin\frac{3x-x}{2}}{\cos 2x}$

$= \dfrac{2\cos\cancel{2}x \times \sin x}{\cos\cancel{2}x}$

$= 2\sin x = R.H.S.$

L.H.S. R.H.S.

Hence proved

10. $\sin(n+1)x \quad \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x$

$\sin A \quad \sin B + \cos A + \cos B$

$\cos(A-B)$

$\cos[(n+1)x - (n+2)x]$

$\cos[nx + x - (nx + 2x)]$

$\cos[\cancel{nx} + x - \cancel{nx} - 2x]$

$\cos -x$

$\cos x$

Hence proved

13. $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

$(\cos 2x + \cos 6x)(\cos 2x - \cos 6x)$

$\left[2\cos\frac{2x+6x}{2}\cos\frac{2x-6x}{2}\right]\left[-2\sin\frac{2x+6x}{2}\sin\frac{2x-6}{2}\right]$

$(2\cos 4x \cos 2x)\cdot(2\sin 4x \sin 2x)$

$(2\cos 4x \sin 4x)\ (2\sin 2x \cos 2x)$

$\therefore 2\sin x \cos x = \sin 2x$

$\sin 8x \sin 4x$

Hence proved

14. $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

$2\sin 4x + (\sin 2x + \sin 6x)$

$2\sin 4x + \left[2\sin\frac{2x+6x}{2}\cos\frac{2x-6x}{2}\right]$

$2\sin 4x + 2\sin 4x \cos 2x$

$2\sin 4x(\cos 2x + 1)$

$2\sin 4x \times 2\cos^2 x = 4\cos^2 x \sin 4x$ Hence proved

16. $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = \dfrac{-\sin 2x}{\cos 10x}$

$$\frac{-2\sin\frac{9x+5x}{2}\ \sin\frac{9x-5x}{2}}{2\cos\frac{17x+3x}{2}\ \sin\frac{17x-3x}{2}}$$

$\Rightarrow \dfrac{-\cancel{2\sin 7x}}{2\cos 10x}\ \dfrac{\sin 2x}{\cancel{\sin 7x}} = \dfrac{-\cancel{2}\sin 2x}{\cancel{2}\cos 10x} = \dfrac{-\sin 2x}{\cos 10x}$

Hence proved

17. $\dfrac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x} = \tan 4x$

$$\frac{2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}{2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}$$

$$\frac{\cancel{2}\sin 4x\ \cancel{\cos x}}{\cancel{2}\cos 4x\ \cancel{\cos x}}$$

$$= \frac{\sin 4x}{\cos 4x} = \tan 4x \quad \text{Hence Proved}$$

18. $\dfrac{\sin x - \sin y}{\cos x+\cos y} = \tan\dfrac{x-y}{2}$

$$\frac{\cancel{2\cos\frac{x+y}{2}}\ \sin\frac{x-y}{2}}{\cancel{2\cos\frac{x+y}{2}}\ \cos\frac{x-y}{2}}$$

$$\frac{\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}} = \tan\frac{x-y}{2}$$

Hence proved

19. $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

$$\frac{2\sin\frac{x+3x}{2}\cos\frac{x-3x}{2}}{2\cos\frac{x+3x}{2}\cos\frac{x-3x}{2}}$$

$$\frac{\cancel{2}\sin 2x\ \cancel{\cos x}}{\cancel{2}\cos 2x\ \cancel{\cos x}}$$

$$\frac{\sin 2x}{\cos 2x} = \tan 2x$$

Hence proved

21. $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

$$\frac{\cos 3x + \cos 4x + \cos 2x}{\sin 3x + \sin 4x + \sin 2x}$$

$$\frac{\cos 3x + 2\cos\frac{4x+2x}{2}\cos\frac{4x-2x}{2}}{\sin 3x + 2\sin\frac{4x+2x}{2}\cos\frac{4x-2x}{2}}$$

$$\frac{\cos 3x + 2\cos 3x\cos x}{\sin 3x + 2\sin 3x\cos x}$$

$$\frac{\cos 3x\,\cancel{(1+2\cos x)}}{\sin 3x\,\cancel{(1+2\cos x)}} = \cot 3x.$$

Hence proved

22: $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$

$\cot x \cot 2x - \cot 3x (\cot 2x + \cot x)$

$\cot x \cot 2x - \cot (2x + x) (\cot 2x + \cot x)$

Using formula

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B} \Rightarrow \cot x \cot 2x \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x} (\cot 2x + \cot x)\right]$$

$\cot x \cot 2x - (\cot 2x \cot x - 1)$

$= 1$ R.H.S.

Hence proved

23. $$\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

L.H.S. $\tan 4x = \tan 2(2x)$

By using formula

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$= \frac{2 \tan 2x}{1 - \tan^2 (2x)}$$

$$= \frac{2\left[\frac{2 \tan x}{1 - \tan^2 x}\right]}{1 - \left[\frac{2 \tan x}{1 - \tan^2 x}\right]^2}$$

$$= \frac{\left[\frac{4\tan x}{1-\tan^2 x}\right]}{\left[1 - \frac{4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

$$= \frac{\left[\frac{4\tan x}{1-\tan^2 x}\right]}{\frac{(1-\tan^2 x)^2 - 4\tan^2 x}{(1-\tan^2 x)^2}}$$

$$= \frac{4\tan x\,(1-\tan^2 x)}{(1-\tan^2 x)^2 - 4\tan^2 x}$$

$$= \frac{4\tan x\,(1-\tan^2 x)}{1+\tan^4 x - 2\tan^2 x - 4\tan^2 x}$$

$$= \frac{4\tan x\,(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$ Hence proved

24. $\cos 4x = 1 - 8\sin^2 x\cos^2 x$

L.H.S. $= \cos 4x = \cos 2(2x)$

R.H.S. $= \cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\sin^2 2x$ $\quad \sin 2A = 2\sin A\cos A$

$= 1 - 2(2\sin x\cos x)^2$

$= 1 - 8\sin^2 x\cos^2 x$

$=$ R.H.S. Hence proved

25. $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18\cos^2 x - 1$

L.H.S. $\cos 6x = \cos 3(2x)$

$\because \cos 3A = 4\cos^3 A - 3\cos A$

$= 4\cos^3 2x - 3\cos 2x$

Using formula $\cos 2x = 2\cos^2 2x - 1$

$= 4[(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)]$

$= 4[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x)^2 + 3(2\cos^2 x)] - 6\cos^2 x + 3$

$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

$=$ R.H.S. Hence proved

Chapter = 3

Trignometric functions

Ques- Evaluate $\sin \frac{7\pi}{4}$

Sol:- $\sin\left[2\pi - \frac{\pi}{4}\right] = -\sin\frac{\pi}{4}$

$= \frac{-1}{\sqrt{2}}$ (Ans)

Ques- Find the value of trignometric ratios if $\cos\left[-\frac{25\pi}{4}\right]$

Sol:-

$= \cos\left[6\pi - \left[-\frac{\pi}{4}\right]\right]$

$= \cos\left[6\pi + \frac{\pi}{4}\right]$

$= \cos\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$ (Ans)

Ques - Find $\sin x$ and $\tan x$, if $\cos x = \frac{-12}{13}$ and x lies in the third quadrant.

Sol:-

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - \left[\frac{-12}{13}\right]^2$

$\sin x = \sqrt{1 - \frac{144}{169}}$

$= \sqrt{\frac{169-144}{169}}$

$$\sqrt{\frac{25}{169}} = \frac{-5}{13}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{-5}{13}}{\frac{-12}{13}}$$

$$= \frac{\cancel{-}5 \times \cancel{13}}{\cancel{-}12 \times \cancel{13}}$$

$$= \frac{5}{12} \text{ (Ans)}$$

Ques - Find all other trigonometric ratios if $\sin x = \frac{-2\sqrt{6}}{5}$ and x lies in quadrant III

Sol:-

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\frac{-2\sqrt{6}}{5}}$$

$$= \frac{-5}{2\sqrt{6}}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\left[\frac{-2\sqrt{6}}{5}\right]^2 + \cos^2 x = 1$$

$$1 - \frac{24}{25} = \cos^2 x$$

$$\cos^2 x = \frac{25-24}{25} = \frac{1}{25}$$

$$\cos x = \frac{-1}{5}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\frac{-1}{5}} = -5$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \frac{-2\sqrt{6}}{\cancel{2}5} \times \frac{-5}{1}$$

$$= 2\sqrt{6}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{2\sqrt{6}} = 2\sqrt{6}$$

<u>Ques</u> - Find the value of cosec 390°

$$\operatorname{cosec} 390^\circ = \operatorname{cosec}(1 \times 360 + 30)$$

$$= \operatorname{cosec} 30^\circ$$

$$= \operatorname{cosec} \frac{\pi}{6} = 2$$

<u>Ques</u> - Prove that $3 \sin \frac{\pi}{6} \sec \frac{\pi}{3} - 4 \sin \frac{5\pi}{6} \cot \frac{\pi}{4} = 1$

$$= 3 \times \frac{1}{\cancel{2}} \times \cancel{2} - 4 \sin\left[\pi - \frac{\pi}{6}\right] \times 1 = 1$$

$$= 3 - \cancel{4} \times \frac{1}{\cancel{2}} \times 1 = 1$$

$$= 3 - 2 = 1$$

$$= 1 = 1$$

L.H.S. = R.H.S. <u>Hence proved</u>

Ques - Find the value of trigonometric ratios if $\cos\left[\frac{39\pi}{4}\right]$

$$\cos\left[10\pi - \frac{\pi}{4}\right]$$
$$= -\cos\frac{\pi}{4}$$
$$= \frac{1}{\sqrt{2}} \text{ (Ans)}$$

Ques - Prove that $\cos 510^\circ \cos 330^\circ + \sin 390^\circ \cos 120^\circ = -1$

$$L.H.S. = \cos 510^\circ \cos 330^\circ + \sin 390^\circ \cos 120^\circ$$
$$= \cos\frac{17\pi}{6} \cos\frac{11\pi}{6} + \sin\frac{13\pi}{6} \cos\frac{2\pi}{3}$$
$$= \cos\left[3\pi - \frac{\pi}{6}\right] \cos\left[2\pi - \frac{\pi}{6}\right] + \sin\left[2\pi + \frac{\pi}{6}\right] \cos\left[\pi - \frac{\pi}{3}\right]$$
$$= -\cos\frac{\pi}{6} \cos\frac{\pi}{6} + \sin\frac{\pi}{6} - \cos\frac{\pi}{3}$$
$$= \frac{-\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times -\frac{1}{2}$$
$$= \frac{-3}{4} - \frac{1}{4} = \frac{-\cancel{4}}{\cancel{4}} = -1$$

L.H.S. = R.H.S. Hence proved

Ques - Prove that $\tan 4x - \cos\frac{3\pi}{2} - \sin\frac{5\pi}{6} \cos\frac{2\pi}{3} = \frac{1}{4}$

$$L.H.S. \; 4 \times (0) - 0 - \sin\left[\pi - \frac{\pi}{6}\right] \cos\left[\pi - \frac{\pi}{3}\right]$$
$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

L.H.S. = R.H.S.

Hence proved

Ques - $\cos 270^\circ$

$$\cos \frac{3\pi}{2} = \cos\left[\pi + \frac{\pi}{2}\right] = \cos\frac{\pi}{2} = 0$$

Ques $\tan 480^\circ$

$$\tan \frac{8\pi}{3}$$

$$= \left[3\pi - \frac{\pi}{3}\right]$$

$$= -\tan\frac{\pi}{3} = -\cot\frac{\pi}{6}$$

$$= -\sqrt{3} \text{ (Ans)}$$

Ques - $\cot 570^\circ$

$$\cot \frac{19\pi}{6}$$

$$= \cot\left[3\pi + \frac{\pi}{6}\right]$$

$$= \cot\frac{\pi}{6} = \sqrt{3} \text{ (Ans)}$$

Ques - $\sin\left[\frac{-25\pi}{4}\right]$

$$\sin\left[\frac{12\pi}{2} + \frac{\pi}{4}\right]$$

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \text{ (Ans)}$$

<u>Ques</u>- Prove that $$\frac{\cos\left[\frac{\pi}{2}+x\right]\sec(-x)\tan(\pi-x)}{\sec(2\pi-x)\sin(\pi+x)\cot\left[\frac{\pi}{2}-x\right]} = -1$$

$$\frac{\cancel{(-\sin x)}\cancel{(\sec x)}\cancel{(-\tan x)}}{\cancel{\sec x}\cancel{(-\sin x)}\cancel{(\tan x)}} = -1$$

Hence proved

<u>Ques</u>- Evaluate $\sin\left[-11\frac{\pi}{3}\right] = \cancel{1}$

$$= \left[7\frac{\pi}{2}+\frac{\pi}{6}\right]$$

$$= -\cos\frac{\pi}{6}$$

$$= \frac{-\sqrt{3}}{2} \text{ (Ans)}$$

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Miscellaneous Exercise ≥

Example 29 Prove that $\cos^2 x + \cos^2\left[x + \frac{\pi}{3}\right] + \cos^2\left[x - \frac{\pi}{3}\right] = \frac{3}{2}$

$$\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

L.H.S.

$$\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right)$$

$$= \frac{1 + \cos 2x}{2} + \frac{1 + \cos 2\left(x + \frac{\pi}{3}\right)}{2} + \frac{1 + \cos 2\left(x - \frac{\pi}{3}\right)}{2}$$

$$\frac{1}{2}\left[1 + \cos 2x + 1 + \cos\left(2x + \frac{2\pi}{3}\right) + 1 + \cos\left(2x - \frac{2\pi}{3}\right)\right.$$

$$\frac{1}{2}\left[3 + \cos 2x + \cos\left(2x + \frac{2\pi}{3}\right) + \cos\left(2x - \frac{2\pi}{3}\right)\right.$$

$$\frac{1}{2}\left[3 + \cos 2x + 2\cos\left(\frac{2x + \cancel{\frac{2\pi}{3}} + 2x - \cancel{\frac{2\pi}{3}}}{2}\right) \cdot \cos\left(\frac{2x + \frac{2\pi}{3} - 2x + \frac{2\pi}{3}}{2}\right)\right.$$

$$\frac{1}{2}\left[3 + \cos 2x + 2\cos 2x \times \cos\frac{2\pi}{3}\right)$$

$$\frac{1}{2}\left[3 + \cos 2x + 2\cos 2x \times -\frac{1}{2}\right)$$

$$\frac{1}{2}\left[3+\cancel{\cos 2x}-\cancel{\cos 2x}\right]$$

$$=\frac{1}{2}\times 3=\frac{3}{2}=R.H.S.$$

<u>Ques1</u> - $2\cos\frac{\pi}{13}\quad\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

L.H.S.

$$2\cos\frac{\pi}{13}\quad\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{5\pi}{13}+\cos\frac{3\pi}{13}$$

$$\left[\cos x+\cos y=2\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2}\right]$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{\frac{5\pi}{13}+\frac{3\pi}{13}}{2}\cos\frac{\frac{5\pi}{13}-\frac{3\pi}{13}}{2}$$

$$=2\cos\frac{\pi}{13}\cdot\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cdot\cos\frac{\pi}{13}$$

$$=2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\cos\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right]$$

$$=4\cos\frac{\pi}{13}\cdot\cos\frac{\pi}{2}\cdot\cos\frac{5\pi}{26}$$

4. $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x-y}{2}$

$\cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$

$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2(\cos x \cos y + \sin x \sin y)$

$= 1 + 1 - 2\cos(x-y)$

$= 2 - 2\cos(x-y)$

$= 2[1 - \cos(x-y)]$

$= 2\left[\cancel{1} - \left(\cancel{1} - 2\sin^2 \frac{x-y}{2}\right)\right]$

$= \cos\theta = 1 - 2\sin^2 \frac{\theta}{2}$

$= 4\sin^2 \frac{x-y}{2}$ R.H.S.

6. $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

L.H.S.

$\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$

Sin

$\sin x + \sin y = 2\sin \frac{x+y}{2} \cdot \cos \frac{x-y}{2}$

$\cos x + \cos y = 2\cos \frac{x+y}{2} \cdot \cos \frac{x-y}{2}$

$2\sin(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)$

Miscellaneous Exercise

If $\tan x = -\frac{4}{3}$, x lies in 2^{nd} quadrant. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\tan\frac{x}{2}$

$$\tan x = \frac{2\tan x/2}{1-\tan^2 x/2}$$

$$-\frac{\cancel{4}^{\,2}}{3} = \frac{\cancel{2}\tan x/2}{1-\tan^2 x/2}$$

$$-2(1-\tan^2 x/2) = 3\tan x/2$$

$$-2 + 2\tan^2 x/2 - 3\tan x/2 = 0$$

$$2\tan^2 x/2 - 3\tan x/2 \cancel{= 0} - 2 = 0$$

$$2\tan^2 x/2 - (4-1)\tan x/2 - 2 = 0$$

$$2\tan^2 x/2 - 4\tan x/2 + \tan x/2 - 2 = 0$$

$$2\tan x/2(\tan x/2 - 2) + 1(\tan x/2 - 2) = 0$$

$$(\tan x/2 - 2)(2\tan x/2 + 1) = 0$$

$$2\tan x/2 + 1 = 0$$

$$\Rightarrow \tan x/2 = -\frac{1}{2} \quad \left[\begin{array}{l}\text{invalid} \\ x/2 \in 1^{st}\text{ quad.}\end{array}\right]$$

$$\tan x/2 - 2 = 0$$

$$\tan x/2 = 2$$

$$\sec^2 x/2 = 1 + \tan^2 x/2$$

$$= 1 + 2^2 = 5$$

$$\sec x/2 = \sqrt{5}$$

$$\cos x/2 = \frac{1}{\sqrt{5}} \text{ Ans}$$

$$\sin^2 x/2 = 1 - \cos^2 x/2 = 1 - \frac{1}{5} = \frac{4}{5} \qquad \sin\frac{x}{2} = \frac{2}{\sqrt{5}}$$

$$\boxed{\sin x/2 = \frac{2}{\sqrt{5}}} \text{ Ans}$$

Ques - $\cos x = -\frac{1}{3}$, x in quadrant III

$\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\tan\frac{x}{2} = ?$

$$\cos x = -\frac{1}{3}$$

$$\cancel{\cos 2x = 1 - 2\cos^2 x - 1}$$

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2} = \frac{1-\left[-\frac{1}{3}\right]}{2}$$

$$= \frac{1+\frac{1}{3}}{2} = \frac{4}{6} = \frac{2}{3}$$

$x \in \text{III}^{rd}$

$$180 \le x \le 270$$

$$\frac{180}{2} \le \frac{x}{2} \le \frac{270}{2}$$

$$90 \le \frac{x}{2} \le 135$$

$x \in \text{II}^{rd}$

$$\sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left[-\frac{1}{3}\right]}{2} = \frac{1-\frac{1}{3}}{2} = \frac{2}{6} = \frac{1}{3}$$

$$\cos\frac{x}{2} = -\frac{1}{\sqrt{3}}$$

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$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\frac{\sqrt{2}}{\sqrt{3}}}{\frac{-1}{\sqrt{3}}}$$

$$= -\sqrt{2}$$

Ans

Ques 2 - $(\sin 3x + \sin x)\sin x + (\cos 3x - \cos x)\cos x = 0$

$\sin 3x \cdot \sin x + \sin^2 x + \cos x \cos 3x - \cos^2 x$
$= \cos 3x \cos x + \sin 3x \sin x - \cos^2 x + \sin^2 x$
$= \cos(3x - x) - (\cos^2 x - \sin^2 x)$
$= \cos 2x - \cos 2x$
$= 0 = 0$

Hence proved

3. $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2\frac{x+y}{2}$

$= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$
$= 1 + + + 2\,(\cos x \cos y - \sin x \sin y)$
$= 2 + 2\cos(x+y)$
$= 2(1 + \cos(x+y)$

$\cos(x+y) = 2\cos^2\left(\frac{x+y}{2}\right) - 1$

$= 2\left(1 + 2\cos^2\frac{x+y}{2} - 1\right)$

$4\cos^2\frac{x+y}{2}$

Hence proved

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5: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos \sin 4x \cos 2x$

$2\sin \frac{x+3x}{2} \cos\left(\frac{x-3x}{2}\right) + 2\sin \frac{5x+7x}{2} \cos\left(\frac{5x-7x}{2}\right)$

$2\sin 2x \cos(-x) + 2\sin 6x \cos(-x)$
$2\sin 2x \cos x + 2\sin 6x \cos x$

$2\cos x\,(2\sin 2x + 2\sin 6x)$

$2\cos x\left(2\sin \frac{2x+6x}{2}\right) \cos\left(\frac{2x-6x}{2}\right)$

$2\cos x\,(2\sin 4x \cos(-2x))$
$4\cos x \sin 4x \cos 2x$

7 $\sin 3x + \sin 2x - \sin x = 4\sin \cos \frac{x}{2} \cos \frac{3x}{2}$

$\sin 3x - \sin x + \sin 2x$

$\sin A - \sin B = 2\cos \frac{A+B}{2} \sin\left(\frac{A-B}{2}\right)$

$2\cos \frac{3x+x}{2} \sin\left(\frac{3x-x}{2}\right) + \sin 2x$

$2\cos 2x \sin x + \sin 2x$
$2\cos 2x \sin x + 2\sin x \cos x$
$2\sin x\,(\cos 2x + \cos x)$
$2\sin x\left(2\cos\left(\frac{2x+x}{2}\right) \cos\left(\frac{2x-x}{2}\right)\right)$

$4\sin x \cos \frac{3x}{2} \cos \frac{x}{2}$

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10.

$$\cos 2x = 1 - \sin^2 x$$
$$\cos 2x = 1 - \left(\frac{1}{4}\right)^2$$

$$\cos^2 x = 1 - \frac{1}{16}$$

$$\cos x = \pm \sqrt{\frac{16-1}{16}}$$

$$\cos = \pm \frac{\sqrt{15}}{4}$$

x lies in 2nd quad.

$$\cos x = -\frac{\sqrt{15}}{4}$$

$$2\sin^2 \frac{x}{2} = 1 - \cos x$$

$$2\sin^2 \frac{x}{2} = \frac{1}{1} + \frac{\sqrt{15}}{4}$$

$$\sin^2 \frac{x}{2} = \frac{4 + \sqrt{15}}{4 \times 2}$$

$$\sin \frac{x}{2} = \pm \sqrt{\frac{(4+\sqrt{15}) \times 2}{8 \times 2}}$$

$$= \pm \sqrt{\frac{8 + 2\sqrt{15}}{4}}$$

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$= 1 + \cos x$

$= \frac{1}{1} - \frac{\sqrt{15}}{4}$

$= \frac{4 - \sqrt{15}}{8}$

$= \sqrt{\frac{4 - \sqrt{15}}{8 \times 2} \times 2}$

$= \sqrt{\frac{8 - 2\sqrt{15}}{4}}$

$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$\frac{\sqrt{8 + 2\sqrt{15}}}{4} \times \frac{4}{\sqrt{8 - 2\sqrt{15}}}$

$\frac{\sqrt{8 + 2\sqrt{15}}}{\sqrt{8 - 2\sqrt{15}}} \times \frac{\sqrt{8 + 2\sqrt{15}}}{\sqrt{8 + 2\sqrt{15}}}$

$\frac{(\sqrt{8 + 2\sqrt{15}})^2}{\sqrt{(8)^2 - (2\sqrt{15})^2}} = \frac{8 + 2\sqrt{15}}{\sqrt{64 - 4 \times 15}}$

$\frac{\cancel{8}^{4} + 2\sqrt{15}}{\cancel{2}}$

$\frac{\cancel{2}(4 + \sqrt{15})}{\cancel{2}}$

$\tan\frac{x}{2} = 4 + \sqrt{15}$ (Ans)